Research statement

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This research statement has two parts: a short overview of my work so far and future directions, and a longer detailed research plan including more concrete conjectures and approaches towards proving them.

Overview

My research is in the fields of group theory, combinatorics and lately also in theoretical computer science. My primary mathematical research interest is expansion phenomena in groups and simplicial complexes. Studying these phenomena lead me to use and develop tools from geometric group theory, algebraic topology, coarse geometry, functional analysis, representation theory and combinatorics. Lately, my interests expanded (no pun intended) and now I am also working on questions of expansion relating to (and arising from) theoretical computer science.

Expansion is easiest to define in the setting of graphs, in which a family of expander graphs is a family of graphs with uniformly bounded degree that are uniformly "highly connected" (being highly connected is measured either by the isoperimetric constant known as the Cheeger constant or by the second eigenvalue of the random walk operator). Expander graphs have strong relations to expansion in groups - for instance, the first explicit construction of expander graphs was given by Margulis using Kazhdan's property (T) (which is an expansion property for groups).

Expander graphs have found many applications in pure mathematics and theoretical computer science. This fruitfulness motivated a (much more recent) development of a new (sub)field of study known as high dimensional expanders, i.e., expansion in simplicial complexes that are the high dimensional counterpart of graphs. High dimensional expanders have attracted much interest lately: a special year was devoted to this subject in the IIAS, a summer cluster on this topic and its connections to computer science was held at the Simons Institute in Berkeley and many papers were written (for instance, Lubotzky's survey paper on this subject [28] written at the end of 2017 is already vastly outdated). Unlike expander graphs, only group-theoretic constructions of high dimensional expanders that have bounded degree with respect to vertices are known (there is a probabilistic model for high dimensional expanders with a weaker notion of bounded degree - see [29]). In other words, there is no known way to choose random simplicial complexes that will exhibit expansion properties and have a strong notion of bounded degree (it was conjectured both by Lubotzky and by Gromov independently that no such model exists). This motivates the philosophy that high dimensional expanders are "better than random" and as such should have powerful applications that are unattainable using random methods.

My work on expansion started with the study of expansion phenomena of groups acting on simplicial complexes. In that setting there is a powerful "local to global" philosophy, sometimes known as "Garland's method" (referring to [17]), that allows one to deduce expansion for the group, given good expansion in the links of a simplicial complex on which it acts. In my work, I have utilized this philosophy and showed how it can be used to deduce various expansion phenomena that were previously unknown (see [20], [33], [36], [37], [38], [39], [43]). These works can be divided into two themes: Improving Garland's method in the Hilbert setting ([20], [36], [37]) and generalizing Garland's method to Banach spaces ([33], [38], [39], [43]). In my detailed research plan below, I describe my future projects with respect to the latter theme. Notably, an ambitious project described below is to prove Banach expansion for $\mathrm{SL}_n(\mathbb{Z})$ when $n > 2$ (or more generally for groups graded by root systems - see definition in [14]) with respect to all uniformly convex Banach spaces. A consequence of successfully completing this project will also give a positive answer to the question (attributed to Margulis), asking whether the Margulis expanders are super-expanders (i.e., expanders with respect to all uniformly convex Banach spaces).

Having studied expansion of groups acting on simplicial complexes, it was natural for me to also study high dimensional expanders. Indeed, my work shows that the same local to global philosophy can be used in the setting of simplcial complexes to prove expansion phenomena associated to spectral expansion such as spectral gaps (see [41]), mixing (see [42]) and convergence of high dimensional random walks (see [26] - joint with Tali Kaufman). Notably, the results in [26] were applied by Anari, Liu, Oveis and Vinzant [3] to prove a famous conjecture of Mihail and Vazirani. There are several research directions that I am pursuing regarding high dimensional expanders, but most notably, in an ongoing work with Tali Kaufman, we are able to recast local testability of codes as a high dimensional expansion phenomenon covering the known examples of such codes and hopefully finding better locally testable codes.

Also, in [25] (which is a joint work with Tali Kaufman), we constructed new examples of high dimensional expanders based on groups with expansion properties. These new constructions were met with much enthusiasm in the computer science community (in particular, the paper was published in STOC), since it gave for the first time an elementary construction of high dimensional expanders (previous examples known as Ramanujan complexes [31], [32] required much theoretical machinery from the theory of Bruhat-Tits buildings and representation theory). We also proved these examples give new examples of expanders with respect to $\mathbb{F}_2$-coefficients (see [24]). Recently, our work as generalized by Friedgut and Iluz [16] to give the first example of regular high dimensional expanders.

The study of expansion also lead me to some "side projects": First, I have studied coarse geometry properties that prevent expansion (see [34], [35]). Second, in my study of Banach property (T), I have developed a new operator theoretical criterion for convergence of alternating projections methods based on a new concept of angles between projections (see [40]). Third, in a joint work with Alex Lubotzky, we prove that my work on vanishing of cohomology with Banach coefficients implies that there is a group which is not approximated with respect to any p-Schatten norm, when $1 < p < \infty$ (the result and more details can be found in [30]).

Detailed Research Plan

Group expansion

Banach expansion for groups graded by root systems. The class of groups graded by root systems was defined in [14]. I will not give the definition here, but give the well-known example of $\mathrm{SL}_n(\mathbb{Z})$ ($n > 2$) which is a group graded by a root system. In [14], it was proven that groups graded by root systems have property (T) and as a corollary it follows that the family of (high rank) simple groups of Lie type are expanders. I conjecture that the results of [14] can be generalized to show that groups graded by root systems have (a strengthened version) of Banach property (T) with respect to the class of all uniformly convex Banach spaces. A consequence of this conjecture is that simple groups of Lie type are new examples of super-expanders. The argument of [14] has two main components - almost orthogonality between subgroups and what I call below a sparsification argument. Generalizing both these components to the Banach setting leads to questions of independent interest detailed below.

Before explaining the generalization needed to the Banach setting, let me give a quick overview on the method I am trying to generalize: The concept of angle between subgroups (or ε-orthogonality) as a tool for proving groups expansion was first introduced by Burger [5] and Dymara and Januszkiewicz [12] and developed in subsequent works of Ershov, Jaikin-Zapirain and Kassabov [13], [22], [14] in the Hilbert setting and by myself in the Banach setting [38], [39]. The basic setup is of a group G generated by compact subgroups $K_1, ..., K_n$ and if the subgroups are sufficiently orthogonal to each other (i.e., they are pairwise ε-orthogonal with ε sufficiently small), then the group expands (i.e., has property (T) or some variation of property (T)). This notion of orthogonality can be thought of a quantification of commutativity: if K_1 and K_2 commute, then they are orthogonal (i.e., 0-orthogonal) and for some constant $\varepsilon > 0$, they are ε-orthogonal if in some sense they are "ε almost commuting". When the subgroups $K_1, ..., K_n$ are not compact, one needs an additional relative property (T) argument passing from expansion with respect to the subgroups $K_1, ..., K_n$ to expansion with respect to compact generating sets of these subgroups.

In order to prove Banach expansion to groups graded by root systems, the argument of [13] needs to be adapted to the Banach setting. I am currently working on such adaptation for proving that the Cayley graphs of $SL_3(\mathbb{F}_p)$ with standard generators are super-expanders. Even achieving this will be a major breakthrough, but a success in this project will shed light on the more general problem.

Higher Dimensional Kazhdan groups. A natural generalization of property (T) is vanishing of higher cohomologies. In [7], a group is defined to be strongly n-Kazhdan is defined as a group that all its $1, ..., n$ cohomologies vanish with respect to any unitary representation. The motivating reference for this subject is the work of Garland [17] that shows that groups acting cocompactly and properly on $(n+1)$-dimensional buildings are strongly n-Kazhdan given that the thickness of the building is large enough. I have several projects in which the question of vanishing of higher cohomologies. Recently, in a work [20] with my student, Zohar Grinbaum-Reizis, we generalized this statement and prove that any group acting cocompactly and properly on a non-thin $(n+1)$-dimensional affine building is in fact strongly n-Kazhdan (this was known before by the work of Casselman [6], but we gave a geometric proof closing the gap in Garland's original argument). In [7], it was asked whether $\mathrm{SL}_n(\mathbb{Z})$ is 2-Kazhdan (when excluding a finite list of finite dimensional representations). In seems that my work with Zohar Grinbaum-Reizis seems to apply here: In [25], I give a simplicial complex on which the Steinberg group $\mathrm{St}_n(\mathbb{Z})$ acts and I hope I can use this simplicial complex to prove vanishing of higher cohomologies for $\mathrm{St}_n(\mathbb{Z})$ (and the vanishing of higher cohomologies for $\mathrm{SL}_n(\mathbb{Z})$). This approach has two problems: First, I do not know if the (universal cover) of the complex of $\mathrm{St}_n(\mathbb{Z})$ is contractible (or at least have vanishing cohomology up to dimension). I conjecture that it is in fact a CAT(0) complex (which will be an interesting result of its own). Second, since the action on the complex is not proper, I need a higher dimensional relative property (T) argument (for instance, a version of relative property (T) for the second cohomology).

High dimensional expanders

Random walks on partite complexes. Random walks on local spectral high dimensional expanders have been a very active field of study lately (e.g., [2], [8], [9], [26] - all from the last couple of years). In a ongoing work with my student Zohar Grinbaum-Reizis we have been able to achieve quantified version of our work on vanishing of cohomology that will improve on many known results regarding random walks on partite local spectral expanders. An n-dimensional complex is called partite if its vertices can be colored using $n+1$ colors such that every n-dimensional face has vertices of all colors (this is the generalization of a bipartite graph when $n=1$). In our work, we are able to provide an analysis on the random walk of such simplicial complexes that for the first time uses the fact that different links have different spectral gap. Namely, preliminary results show that we are able to improve of three types of results:

1. My results regarding "spectral descent" in [41] (now also know as the "trickling down Theorem").
2. Results regarding up-down high dimensional random walks of [2], [9], [26].
3. Results regarding the "swap walk" in [8].

Expansion with $\mathbb{F}_2$ coefficients. In generalizing the notion of graph expansion to simplicial complexes, two approaches were taken: First, generalizing the notion of the spectral gap - this can be done in several ways, but the most popular today is considering local spectral expansion, i.e., spectral expansion in the links. The second approach that originated in the work of Gromov [21] and Linial and Meshulam [27] is generalizing to notion of the Cheeger constant and considering expansion with respect to $\mathbb{F}_2$ coefficients called coboundary expansion. However, it was shown that even our best example of high dimensional expanders (i.e., Ramanujan complexes) do not always have this property and there is currently no known construction of a family of bounded degree high dimensional expanders that are (uniformly) coboundary expanders. This lead to the definition of a weaker notion of $\mathbb{F}_2$ expansion known as cosystolic expansion (see definitions in [15]) and it was shown in [23] (based on [11]) that cosystolic expanders have a the strong expansion phenomenon of have the topological overlapping property (before [23], this property was only known for coboundary expanders). In [15], Evra and Kaufman gave a method for proving cosystolic expansion and showed that Ramanujan complexes are cosystolic expanders. Their method relied on two components, first local spectral expansion and second coboundary expansion of the links. In recent work with Tali Kaufman, we started to study the role of symmetry in the context of coboundary expansion. We already proved [24] that one can prove coboundary expansion for symmetric simplicial complexes based on bounding their high dimensional radius. We then showed that for 2-dimensional simplicial complexes this high dimensional radius can be bounded using the (Dehn function of the) group presentation. This allowed us to show that our construction in [25] also gives new examples of 2-dimensional coboundary and cosystolic expanders. Generalizing this result for dimension larger than 2 is still open - see more details below. It is interesting to note that there are new developments [19] that show that $\mathbb{F}_2$ expansion is useful for tackling problems in theoretical computer science.

Connection to locally testable codes. A linear binary code $C \subseteq \mathbb{F}_2^n$ is a linear subspace. The idea behind error-correcting codes is that $C \subseteq \mathbb{F}_2^n$ are the allowed code words and when $\underline{c} \in C$ is send through a noisy channel, some of its bits may be corrupted (the entries of $\underline{c}$ are called bits). A sequence of linear codes $C_n \subseteq \mathbb{F}_2^n, n \rightarrow \infty$ are called locally testable (LTC), if there constants $\delta > 0, q \in \mathbb{N}$ independent of n and a probabilistic oracle algorithm that when given (a possibly corrupted) word $\underline{c}' \in \mathbb{F}_2^n$, quarries it in q bits (randomly), accepts with probability 1 is $\underline{c}' \in \mathbb{F}_2^n$ and rejects with probability $> \frac{1}{2}$ if

$$\frac{\min_{\underline{c} \in C_n} \|\underline{c} - \underline{c}'\|_1}{2^n} \geq \delta.$$

In an ongoing project with Tali Kaufman, we were able to show that local testability arises from a high dimensional expansion phenomenon (another connection was showed in [10], but our work is fundamentally different). Currently, our machinery is already able to improve some known results regrading known LTC codes (specifically, regrading affine invariant codes). The most ambitious goal of this project is to use these new ideas to construct LTC's with bounded rate and distance (such LTC's are yet unknown to exist).

New examples of groups acting on complexes

In [25], Tali Kaufman and myself showed that given a group of elementary matrices (or a Steinberg group) over any ring algebra there is a natural way (using coset geometries) to associate to it a simplicial complex on which the group acts. This was done as follows: given a group G with subgroups $K_0, ..., K_n$, there are certain axioms on $G, K_0, ..., K_n$ coming from the study of coset geometries that give rise to a partite (i.e., colorable) complex $X(G; K_0, ..., K_n)$ on which G acts. It is important to note that the links of $X(G; K_0, ..., K_n)$ arise from the subgroups in a similar manner, e.g., the link of a vertex with color 0 is $X(K_0; K_0 \cap K_1, ..., K_0 \cap K_n)$.

In [25], we showed that for $G = \mathrm{EL}_{n+1}(R[1, t_1, ..., t_l])$ (or $G = \mathrm{St}_{n+1}(R[1, t_1, ..., t_l])$), we can find subgroups $K_0, ..., K_n$ such that these axioms hold (R here denotes a unital ring, EL denotes the group of elementary matrices and St denotes the Steinberg group). Example of a family of high dimensional expanders was constructed by passing to quotients in the coefficients (yielding a finite group acting on a finite complex), explicitly our example is $\{X(\mathrm{EL}_{n+1}(\mathbb{F}_p[t]/\langle t^s \rangle); K_0, ..., K_n)\}_{s \in \mathbb{N}, s>4}$. Since the theory of coset geometry was developed in relation to the classification of finite simple groups, it was not studied in relation to infinite groups or expansion and in [25], we gave a new axiom on $K_0, ..., K_n$ that implies spectral expansion (and showed that is it fulfilled in our examples). This idea was picked up by Friedgut and Iluz [16] who used it to construct new examples of highly regular high dimensional expanders.

Below are several directions that I am working on regarding these complexes.

Cohen-Macaulayness. A simplicial complex X is called Cohen-Macaulay over $\mathbb{F}$ if for every simplex $\sigma \in X$, $\widetilde{H}^i(\text{link of } \sigma, \mathbb{F}) = 0$ for every $i < \dim(\text{link of } \sigma)$. In particular, if X is n-dimensional and Cohen-Macaulay over $\mathbb{F}$, then $\widetilde{H}_i(X, \mathbb{F}) = 0$ for every $i < n$. Let $G, K_0, ..., K_n$ be as above and assume that the axioms for constructing $X = X(G; K_0, ..., K_n)$ hold (then X is pure n-dimensional). We ask on what conditions is $X(G; K_0, ..., K_n)$ Cohen-Macaulay. There are two motivations for the question: First, knowing that X is Cohen-Macaulay will allow me to compute the cohomology of G using the equivariant cohomology of X. Second, a quantitative version of Cohen-Macaulay over $\mathbb{F}_2$ (namely bounding the filling constants and thus bounding the high dimensional radius defined in [24]) will allow me to use my result in [24] in order to prove that X is a coboundary expander. In the literature (under weaker assumptions on $G, K_0, ...K_n$) there is a condition for X being simply connected basically saying that G can be presented using generators in $K_0 \cup ... \cup K_n$ such that every relations appears inside one of the subgroups $K_0, ..., K_n$ (see for instance [18], [1] and a quantitative version by Kaufman and myself in [24]). I conjecture that this condition can be generalized to imply Cohen-Macaulayness as follows: the group G should be generated by $\{\bigcap_{0 \leq i \leq n, i \neq j} K_i : 0 \leq j \leq n\}$ and presented such that all the relations should appear in some $\bigcap_{0 \leq i \leq n, i \neq j_1, j_2} K_i$ where $0 \leq j_1 < j_2 \leq n$. I further conjecture that a quantitative version for $\mathbb{F}_2$ can be attained through the Dehn function of that presentation. Proving this, will allow me to invoke my results regarding the Dehn function of the links in [24] and proving that all the links in the construction described above are coboundary expanders.

Coboundary and cosystolic expansion. Using the result of [15], in order to prove that our new examples in [25] of high dimensional spectral expander are also cosystolic expanders (which is the weak notion of $\mathbb{F}_2$ mentioned above), it is enough to prove that their links are coboundary expanders (which is the strong notion of $\mathbb{F}_2$ mentioned above). As noted in the previous paragraph, I conjecture that this should amount to studying the presentation of the complexes arising from the subgroups and in our examples some of the work studying these presentations was already done in [4]. In particular, in [24], even without using the conjecture regarding Cohen-Macaulayness mentioned above, but only quantifying the work on simple connectedness in [1], we will be able to prove that our examples give new 2-dimensional cosystolic expanders.

Other group-theoretic directions. The fact that a group act on a simplicial complex opens the door of studying this group using the complex. Moreover, since my examples in [25] mimics the classical situation of a group acting on a building, I hope to produce some analogue results to those achieved by the theory of buildings. One example of this approach was already mentioned above - exactly as buildings are used to prove vanishing of higher cohomology, I already mentioned above a potential use of [25] to prove that $\mathrm{SL}_n(\mathbb{Z})$ is strongly $(n-1)$-Kazhdan. Let me mention another possible application in that spirit: affine buildings have a very rich boundary theory that can be used to study the groups acting on them (for instance to find the Poisson boundary of the group). It is interesting to see if there is an analogous boundary theory for our general construction (over any ring).

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